

# Two-Dimensional Separating Turbulent Boundary Layers

W. H. Schofield\*

*Aeronautical Research Laboratories, Melbourne, Australia*

It is shown that mean flow similarity based on a velocity scale related to the maximum shear stress (Schofield-Perry similarity) can accurately describe detached two-dimensional turbulent boundary layers provided the origin of the similarity is relocated on the zero velocity streamline in the detached flow. In support of this, data from several different experiments are analyzed and presented. The failure of the standard logarithmic law to accurately describe flow close to the wall in a separating layer is discussed. It is argued that the failure of the logarithmic law is related to the nature of turbulent separation, which is not an event but a process in which the proportion of intermittent flow reversal near the wall gradually increases with distance downstream and therefore, the mean velocities measured within the detachment region contain a proportion of reversed flow which does not follow standard wall similarity. Experimental evidence supports another proposition that a two-dimensional turbulent boundary layer detaches with a universal mean profile shape which is accurately described by Schofield-Perry similarity. After detachment, the outer separated layer shows only small development, so that reattachment occurs with only a slightly different universal mean profile shape.

## Nomenclature

$B$	= Schofield-Perry integral layer thickness = $2.86 \delta^* U_1 / U_s$
$c_f'$	= skin-friction coefficient = $\tau_w / \frac{1}{2} \rho U_1^2$
$D$	= thickness of mean backflow
$H$	= shape factor, = $\delta^* / \theta$
$L$	= distance of $\tau_m$ from origin of profile
$u$	= mean velocity in $x$ direction
$u_\tau$	= friction velocity, = $(\tau_w / \rho)^{1/2}$
$u', v'$	= fluctuating velocities in the $x$ and $y$ direction, respectively
$U_1$	= freestream velocity
$U_s$	= Schofield-Perry velocity scale
$U_m$	= maximum stress velocity, = $(\tau_m / \rho)^{1/2}$
$U_{mp}$	= maximum stress velocity incorporating quadratic stress terms
$U_N$	= maximum velocity in reversed flow profile
$x$	= distance in the main flow direction
$y$	= distance normal to the flow
$\gamma_p$	= fraction of time flow is in the downstream direction
$\delta$	= total boundary-layer thickness
$\delta^*$	= boundary-layer displacement thickness
$\nu$	= kinematic viscosity of the fluid
$\rho$	= fluid density
$\theta$	= boundary-layer momentum thickness
$\tau_w$	= wall shear stress
$\tau_m$	= maximum in shear stress profile

## Subscript

$D$  = measured from the mean dividing streamline

## Superscript

$(\bar{\quad})$  = time-averaged quantity

## Introduction

**T**URBULENT boundary-layer separation from a surface is an important problem because it is usually responsible for setting an upper limit to the performance of aerodynamic devices. As maximum performance usually occurs very close to the separation condition, an ability to predict boundary-layer separation has been, and remains, a major aim of fluid mechanics research. The subsequent problem of developing an ability to describe a separated boundary-layer flow is less pressing, but is still of considerable practical interest in predicting off-design performance and transitory separated flow behavior of aerodynamic components.

Although the topic is hardly new, there has been only limited progress made in our ability to analyze separation. A central reason holding back development in the field is that until fairly recently the experimental data on separation was of poor quality. In particular, observations by different workers of the "separation point" were difficult to compare as this point was variously defined. Also, the experimental detection of the "separation point" was suspect as standard instrumentation was inaccurate near the detachment and surface flow indicators were uncalibrated against the flow parameters. More recent data, gathered by more careful use of old techniques, are an improvement on earlier data, but the important step forward in this field has been the high-quality, highly detailed measurements made with laser Doppler anemometry.<sup>1-4</sup> Because the technique can be highly accurate and nonintrusive, laser Doppler anemometry is ideal for studying intermittently reversed flow, which is the essence of turbulent boundary-layer separation from a faired surface. Accompanying these experiments has been a determined effort<sup>5</sup> to establish generally agreed definitions of the several stages of detachment during turbulent boundary-layer separation. Such an effort is essential because the plethora of definitions of separation used has bedeviled work on the problem.

No single turbulence closure hypothesis available today can model the complete separating layer, i.e., can model layer development as an attached layer in an adverse pressure gradient as well as a detaching and fully detached layer. For this reason, zonal models using a series of relationships to describe the different phases of the flow have been recommended.<sup>6</sup> In this paper, a proposal is developed for a single

Received July 15, 1985; revision received Dec. 23, 1985. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1986. All rights reserved.

\*Senior Principal Research Scientist, Aero Propulsion Division.

similarity description of all (mean) forward flow throughout a two-dimensional separating layer. This unified description is then related to stages in the detachment process. The work leads to a detachment criterion based on mean profile shape.

### Similarity Relations for Separation Layers

The wall/wake similarity of Coles<sup>7,8</sup> is remarkably robust in describing attached turbulent boundary-layer profiles that have developed under a wide range of flow conditions, even including discontinuous changes in boundary conditions.<sup>9</sup> This has encouraged authors to assume its validity right up to the point of detachment in a separating layer. It has even been modified to describe detached profiles with significant mean backflow and negative wall shears.<sup>10,11</sup> However, Simpson observed in both his experiments<sup>1,2</sup> that the usual logarithmic law of the wall was accurate only up to the point at which intermittent flow reversals first appeared in the flow. This has recently been confirmed by Thompson and Whitelaw<sup>11</sup> in a different experiment.

Turbulent boundary-layer separation from a faired surface is not an event, but rather a process developing over an extended length of flow. The process is typified by the flow near the wall spending an increasing proportion of time in (intermittent) backflow as an observer moves downstream through the detachment zone. The intermittency is caused by three-dimensional elements of backflow appearing and disappearing in a random manner near the wall. The appearance, growth rate, and lateral position of these elements is unpredictable and thus there are, within the detachment zone, multiple instantaneous detachment lines that change shape rapidly and continuously.<sup>12</sup> As the detachment process develops downstream, the magnitude and duration of these flow reversal cells increase and the flow becomes increasingly three-dimensional. Simpson et al.<sup>3</sup> observed that there was negligible turbulent energy production within the backflow and hence the classical wall similarity (which depends on a balance between turbulent energy production and dissipation) does not apply to the backflow. Thus, in a region having periods of flow reversal, the similarity relations applying to attached flow will not accurately describe the mean profile because the mean velocity will contain a component of reversed flow that scales on different variables. Flow reversals first occur near the wall and then, as the detachment develops with downstream distance, increase in frequency and extend outward from the wall. Thus, deviation from the logarithmic law starts near the wall and then extends upward through the layer with distance downstream. This is illustrated in Fig. 1 with some mean velocity profiles taken by Simpson et al.<sup>2</sup> and plotted on Clauser<sup>18</sup> coordinates. Initially, the fully attached layer (at  $x=118.5$ ) gives an excellent fit to one of the standard Clauser lines. However, further downstream ( $x=120.5$ ), the flow nearest the wall develops flow reversals and small deviations from the logarithmic law appear. The size and extent of these deviations increase with increasing reversed flow content in the profiles until the "best" standard Clauser line is tangent to, but can no longer be described as correlating, the mean flow. Lines that do correlate the mean flow are also shown in Fig. 1, but have gradients grossly different from the standard Clauser lines.

The detachment zone is thus a zone of transition of the wall flow, from the standard law of the wall for no flow reversals ( $\gamma_p=1$ ) to some different distribution of velocity for continuous backflow ( $\gamma_p=0$ ). At a recent colloquium,<sup>5</sup> definitions for four states of detachment within the detachment zone were agreed upon. They are incipient detachment where  $\gamma_p$  near the wall is 0.99, intermittent transitory detachment at  $\gamma_p$  near the wall of 0.8, transitory detachment at  $\gamma_p$  near the wall of 0.5, and detachment where the time mean wall stress is zero,  $\bar{\tau}_w=0$ . The final two states are closest to the older, vaguer concept of "separation" and the positions at which they occur approximate the "separation point."

Figure 1 shows that, if the standard law of the wall is used to determine the wall shear stress of a profile in a region with intermittent flow reversal, the resulting value of the velocity scale  $u_\tau$  will be inaccurate. Now, if  $u_\tau$  is inaccurate, then so is the entire wall/wake description of the layer because, in Coles' formulation,  $u_\tau$  is the scaling parameter for not only the wall layer but also the outer wake region. To obtain accurate values of wall shear from such profiles, some account has to be made of the reversed flow contamination effects on the measured mean velocities. This has not as yet been attempted, but even if it would be successfully accomplished, the wall/wake model would still not be the best vehicle on which to base an analysis of boundary-layer separation. This is because turbulence measurements suggest that the wall shear in the detachment region, being very small and rapidly approaching zero, has little effect on mean flow development. Flow development in a detachment region is more likely to be governed by the maximum shear stress. Evidence for this comes from experiments by Bradshaw<sup>13</sup> and Simpson et al.<sup>1,3</sup> showing 1) that adverse pressure gradient boundary layers near detachment are dominated by large-scale turbulent structures in the outer layer providing most of the total turbulent energy; and 2) that these large turbulent structures (and other flow parameters) scale with  $U_m$ , which is directly related to the local maximum shear stress, rather than to  $u_\tau$ . Finally, Schofield and Perry<sup>14</sup> (see also Ref. 15) have developed a similarity description for mean velocity profiles that uses a velocity scale based on the local maximum shear stress. This similarity scheme has been shown to be accurate for flows in moderate-to-strong adverse pressure gradients,<sup>16,17</sup> including the attached flow immediately preceding separation.<sup>1,2</sup> As a scaling velocity related to  $\tau_m$  will be unaffected by intermittent or even mean flow reversals near the wall, an analysis of

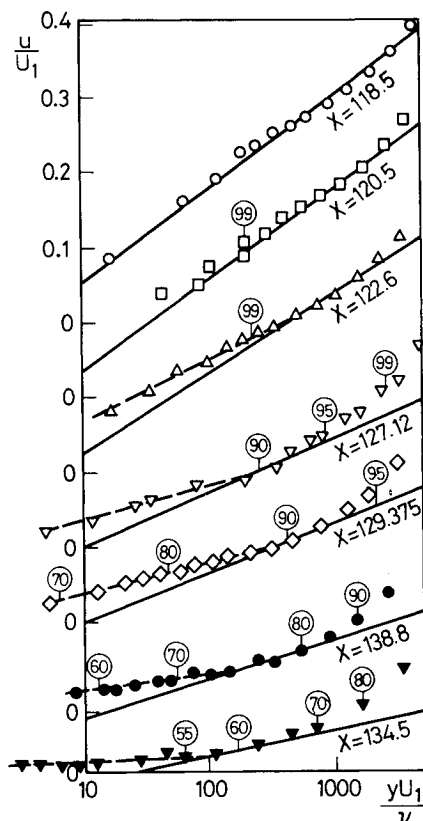


Fig. 1 Mean velocity near the wall in a detaching layer (data of Simpson et al.<sup>2</sup>); numbers in circles give local values of  $\gamma_p$ , — standard Clauser<sup>18</sup> law of the wall lines, — line of best fit to wall flow.

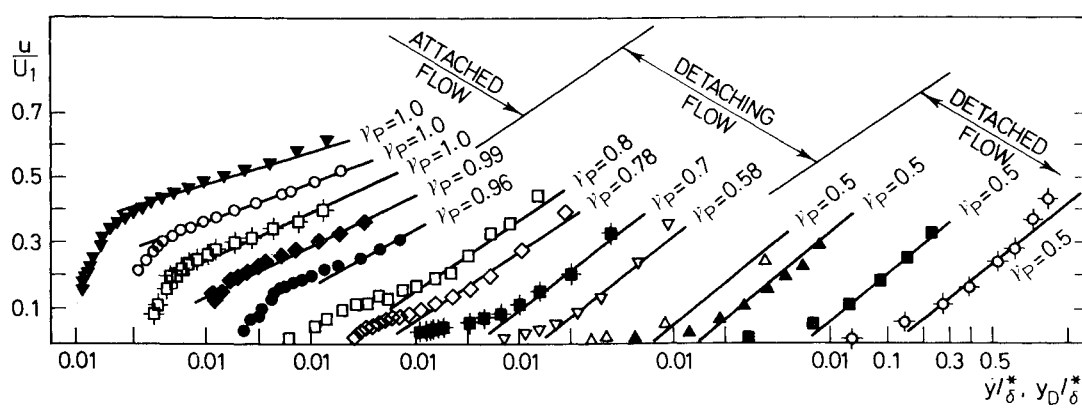


Fig. 2 Half-power distributions of mean velocity in a separating layer (data of Simpson et al.,<sup>2</sup> laser anemometer measurements only).

- ▼  $x = 106.7$ ,  $U_s/U_1 = 0.86$     ○  $x = 112.4$ ,  $U_s/U_1 = 0.75$     ◇  $x = 129.4$ ,  $U_s/U_1 = 1.09$     ■  $x = 131.9$ ,  $U_s/U_1 = 1.20$   
 ◐  $x = 118.5$ ,  $U_s/U_1 = 0.86$     ◆  $x = 120.5$ ,  $U_s/U_1 = 0.91$     ▽  $x = 134.5$ ,  $U_s/U_1 = 1.23$     △  $x = 138.8$ ,  $U_s/U_1 = 1.27$   
 ●  $x = 122.6$ ,  $U_s/U_1 = 0.96$     □  $x = 127.1$ ,  $U_s/U_1 = 1.10$     ▲  $x = 144.9$ ,  $U_s/U_1 = 1.29$     ■  $x = 156.4$ ,  $U_s/U_1 = 1.25$   
 ◐  $x = 170.0$ ,  $U_s/U_1 = 1.22$     — Eq. (2).

detachment using the Schofield and Perry relations should involve only an extension of the analysis developed for fully attached flow.

The Schofield-Perry defect law for the mean velocity is

$$\frac{U_1 - u}{U_s} = 1 - 0.4 \left( \frac{y}{B} \right)^{1/2} - 0.6 \left( \frac{\pi y}{2B} \right) \quad (1)$$

where the integral layer thickness  $B$  is given by  $B = 2.86 \delta^* U_1 / U_s$ . Near the wall, Eq. (1) takes the half-power form

$$\frac{u}{U_1} = 0.47 \left( \frac{U_s}{U_1} \right)^{3/2} \left( \frac{y}{\delta^*} \right)^{1/2} + 1 - \frac{U_s}{U_1} \quad (2)$$

The velocity ratio  $U_s/U_1$  can be determined from a measured profile by adapting Clauser's<sup>18</sup> methodology to Eq. (2). (See Ref. 15 and Fig. 2.) The velocity scale  $U_s$  is related to the maximum shear stress  $\tau_m$  by the relation

$$U_s/U_m = 8.0 (B/L) \quad (3)$$

which is an interesting equation as it relates the mean flow parameters to the turbulent flow parameters. This similarity description applies to adverse pressure gradient layers in which  $\tau_m \geq 3/2 \tau_w$  and accurately describes the outer 90–95% of a profile. For fully attached layers, the inner wall matching condition is given by the usual logarithmic law of the wall.

Intuitively, it seems likely that the validity of this similarity defect law description for two-dimensional flow will not cease abruptly at separation, but will continue to be accurate for the outer portion of the layer where the large-scale coherent turbulence structure continues to determine the mean flow profile. Downstream of detachment, it is unlikely that the defect law will describe the mean flow adjacent to the wall, because the large-scale coherent structures, which generate the mean velocity defect law, are contained in the forward-flowing outer portion of the layer riding over the (mean) reversed flow.<sup>2,3,19</sup> Thus, some part of the backflow region will need to be excluded from the similarity flow description by positioning the origin of the defect law away from the wall for flow downstream of detachment. An obvious choice for the position of this origin is the mean dividing streamline; this choice is conceptually satisfying in that it forms a continuum with the defect law origin for upstream attached flow. It also accords with the turbulence structure that has been reported for a detached layer. Simpson et al.<sup>3</sup> observed that the large-scale coherent structure in

the outer layer was little changed by detachment and dominated the flow from the outer edge of the layer down to the region near the dividing streamline. Large turbulent structures in the outer flow intermittently penetrated below the dividing streamline, where they are modified by the wall to produce backflow. Placement of the origin on the dividing streamline thus puts all the large-scale outer throughflow within the ambit of the defect law, but excludes the wall region into which the outer structure intermittently penetrates and is modified by the wall to produce the backflow.

This proposal was tested against a range of data from two-dimensional detaching and fully detached layers. Initially, the profiles were plotted on half-power coordinates; an example of this is shown in Fig. 2. These are typical results and illustrate a number of findings. First, half-power distributions in velocity exist in both detaching and fully detached profiles and these distributions agree in both position and slope with the family of lines given by Eq. (2) for different values of velocity ratio. Second, as detachment is approached, the outer similarity extends downward toward the wall. Third, there is a steady progression in the slopes of the half-power lines up to detachment, which implies a steady increase in the velocity scale ratio  $U_s/U_1$ . After detachment (where  $y$  is now measured from the dividing streamline), the velocity scale ratio increases, but only very slightly. The data also show that there is a transition in the wall matching profile. It commences as soon as  $\gamma_p < 1$  and is complete at transitory detachment ( $\gamma_p = 0.5$ ). During the transition, the second derivative of  $u/U_1$  with respect to  $y/\delta^*$  changes from negative to positive. In Fig. 2, the detached profiles have been marked with  $\gamma_p = 0.5$  to accord with the condition of  $u = 0$  on the dividing streamline. However,  $u = 0$  and  $\gamma_p = 0.5$  coincide only if the intermittent upstream and downstream flows are statistically similar. Recent work by Wei and Sato<sup>12</sup> does show that the two conditions coincide with good accuracy in a detached layer.

Using the velocity scale ratio determined with graphs similar to Fig. 2, mean profiles from a number of experiments were plotted on Schofield-Perry coordinates; see Figs. 3–8. Simpson's data, which is probably the most reliable, are shown in Fig. 3. The profiles early in the detachment process ( $\gamma_p \sim 1$ ) agree with Eq. (1) within the scatter limits of the data used to formulate the relation for attached flow.<sup>14</sup> Profiles further along the detachment process show some points falling on or just outside the scatter limits for attached flow data. These deviations can be only partially attributed to the forward flow being contaminated with dissimilar reverse flow because many deviations of the data

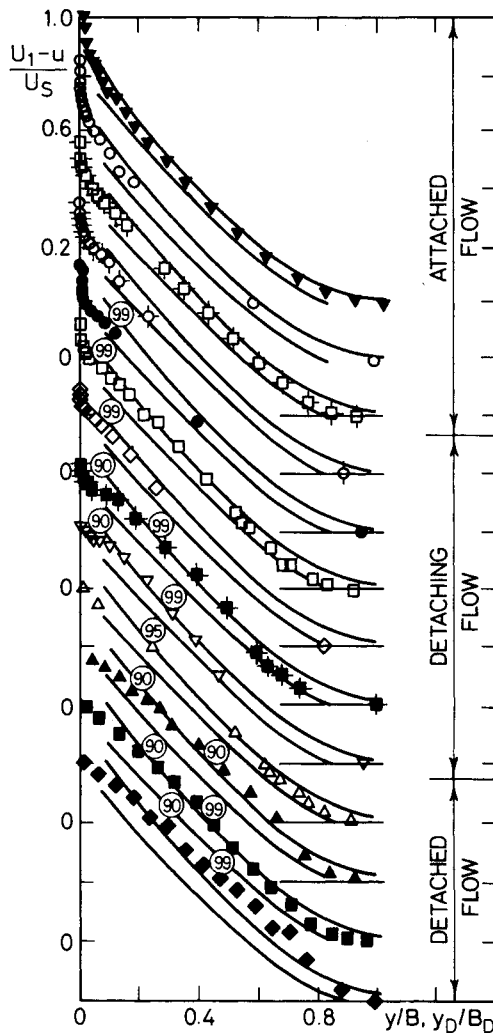


Fig. 3 Mean velocity similarity in a separating layer: data as for Fig. 2, — scatter limits of attached two-dimensional data used to establish Eq. (1) in Ref. 14, numbers in circles give local values of  $\gamma_p$ .

occur in regions where  $\gamma_p = 1$  (for instance, see the last profile in Fig. 3). The progressive deterioration of the correlation with increasing distance downstream can more probably be attributed to the increasing three-dimensionality of the detached flow. As the Schofield-Perry similarity relations were derived from two-dimensional data, they can be expected to have good accuracy only for layers that are closely two-dimensional. In the experiment that produced the data of Fig. 3, in spite of the fact that considerable care was taken to try to maintain two-dimensional flow, three-dimensional flows of increasing strength were noted downstream of detachment and, at the last station, "no nominally two-dimensional flow remained" (Ref. 2, p. 30). Therefore, it is not surprising that the Schofield-Perry defect law does not give a highly accurate description of this profile ( $x = 170$ , Fig. 3). Results of an earlier experiment by Simpson et al.<sup>1</sup> are similar (see Fig. 4), although the quantity of data is much less.

The degree of deviation from the standard correlation will probably be governed by the nature of the flow and the overall flow geometry. Both flows of Simpson et al. developed in a large-aspect-ratio duct. The separated flows did not reattach and there was little to inhibit the development of random three-dimensional movements across the flow. Consequently, the largest deviation from the Schofield-Perry relation is likely to be found in flows like Simpson's. Fairlie's flow (Fig. 5) also developed in a large-aspect-duct,

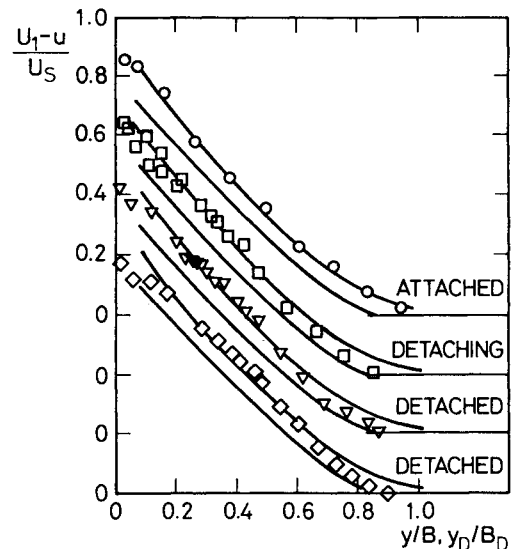


Fig. 4 Mean velocity similarity in a separating layer (data of Simpson et al.,<sup>1</sup> — scatter limits for attached data).

- $x = 103.8$ ,  $\gamma_p = 1.0$ ,  $U_s/U_1 = 0.68$
- $x = 124.3$ ,  $\gamma_p = 0.9$ ,  $U_s/U_1 = 1.05$
- ▽  $x = 139.1$ ,  $\gamma_p < 0.5$ ,  $U_s/U_1 = 1.20$
- ◇  $x = 157.1$ ,  $\gamma_p < 0.5$ ,  $U_s/U_1 = 1.28$

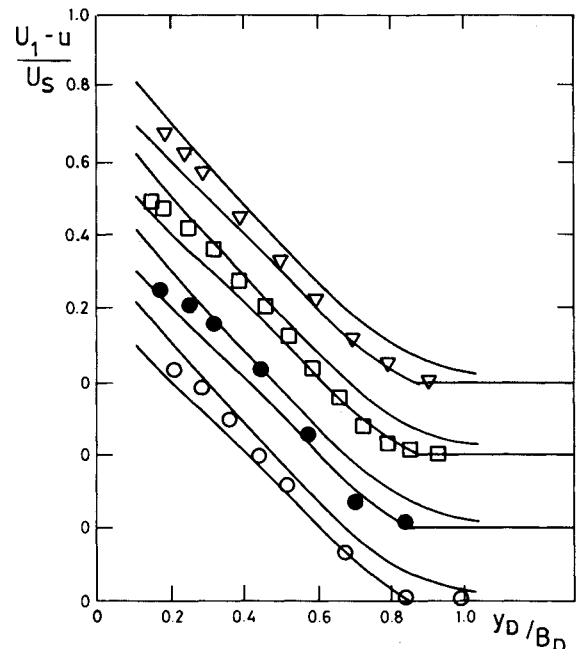


Fig. 5 Mean velocity similarity in a detached layer (data of Fairlie<sup>31</sup> flow 1; see also Perry and Fairlie<sup>30</sup>): ▽  $x = 1.115$  m, □  $x = 1.210$  m, ●  $x = 1.295$  m, ○  $x = 1.402$  m, — scatter limits for detached data. Data points near the dividing streamline have been rejected due to high uncertainty in hot-wire measurements of mean velocity for  $\gamma_p$  near 0.5.

but in this case the flow geometry caused the separated layer to reattach a relatively short distance downstream of detachment. This constraint produced a stabilizing influence on the separated flow and resulted in smaller deviations from Eq. (1).

For separated flows of a quite different nature, the data of Seddon,<sup>20</sup> Schofield,<sup>21</sup> and Delery<sup>22</sup> (Figs. 6–8) were analyzed. In these cases, the separating pressure gradient was very steep as it was generated by a normal shock wave. In all these cases, the layers reattached downstream of detachment.

Equation (3) relates the similarity velocity and length scales of the (outer) mean flowfield ( $U_s$  and  $B$ ) to those of the large-scale turbulence structure ( $U_m$  and  $L$ ) for two-dimensional attached flow. Hence, it provides an integral check on whether a profile conforms with Schofield-Perry similarity. Such checks can be made only with data containing accurate shear stress profiles; therefore, for separating flows we are restricted to the data of Simpson and Delery. It was pointed out by Simpson et al.<sup>1</sup> that for flows approaching detachment,  $U_m$  in Eq. (3) should be replaced by

$$U_{mp} = \left[ -\overline{u'v'} + \int_y^\infty \frac{\partial(\overline{u'^2} - \overline{v'^2})}{\partial x} dy \right]^{1/2}$$

In the original Schofield-Perry analysis of attached layers, the quadratic turbulence terms were neglected as making negligible contributions to the shear stress. However, in detaching, detached, and reattaching layers they can make a small contribution to the stress. These terms have been included in the analysis of Simpson's data, but not in Delery's as there is insufficient detail published in his paper to calculate them. [The value of  $\partial(\overline{u'^2} - \overline{v'^2})/\partial x$  was evaluated using the method described in Ref. 23.]

The reduced data presented in Fig. 9 agree fairly well with Eq. (3), but are more scattered than that for attached layers. As there is no particular trend to the increased scatter, this may simply reflect the increased inaccuracies in measuring (integral) quantities in highly intermittent detached flows.

This analysis and the previously presented correlations show that the similarity scheme based on  $\tau_m$  can, with very little modification, be usefully extended to describe fully detached layers with small error. The similarity scheme is robust and is also compatible with, and to an extent complementary to, the logarithmic law of the wall—robust

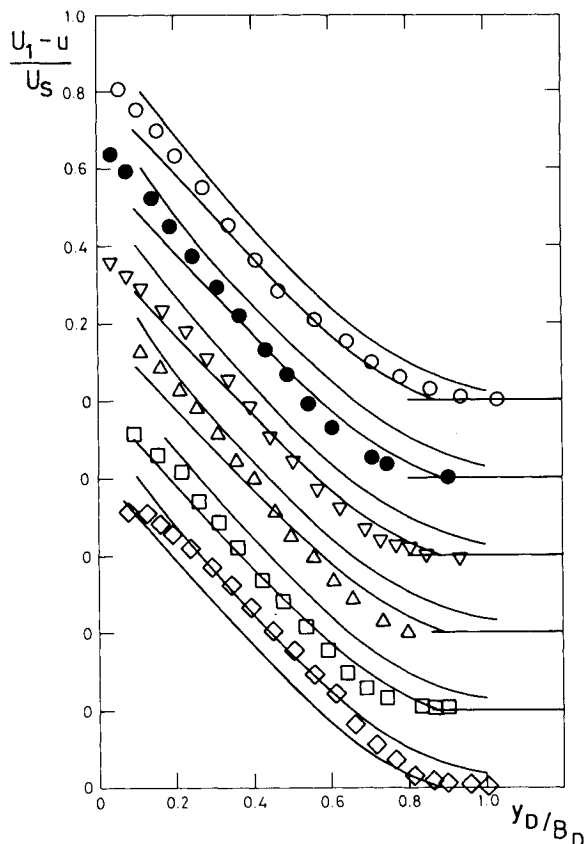


Fig. 6 Mean velocity similarity in a detached layer (data of Seddon<sup>20</sup>, basic interaction):  $\circ$  profile 3,  $\bullet$  profile 4,  $\nabla$  profile 5,  $\Delta$  profile 6,  $\square$  profile 7,  $\diamond$  profile 8, — scatter limits for attached data.

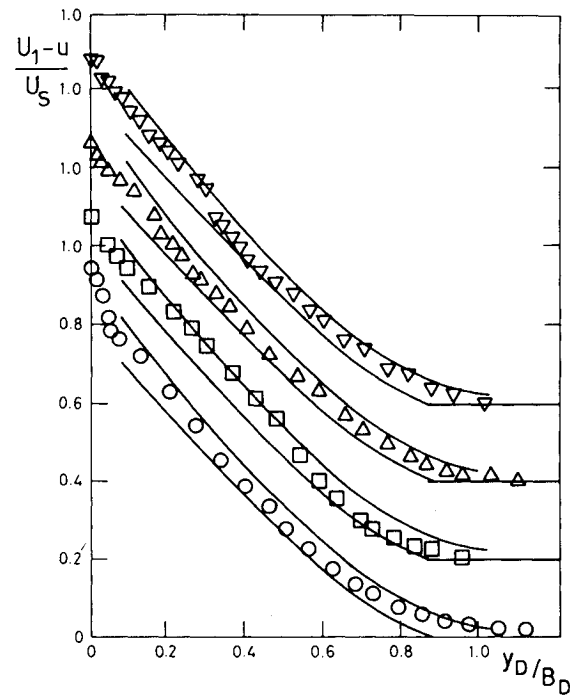


Fig. 7 Mean velocity similarity in a detached layer (data of Schofield<sup>21</sup>):  $\nabla$   $x=0.237$  m,  $\Delta$   $x=0.249$  m,  $\square$   $x=0.263$  m,  $\circ$   $x=0.276$  m, — scatter limits for attached data.

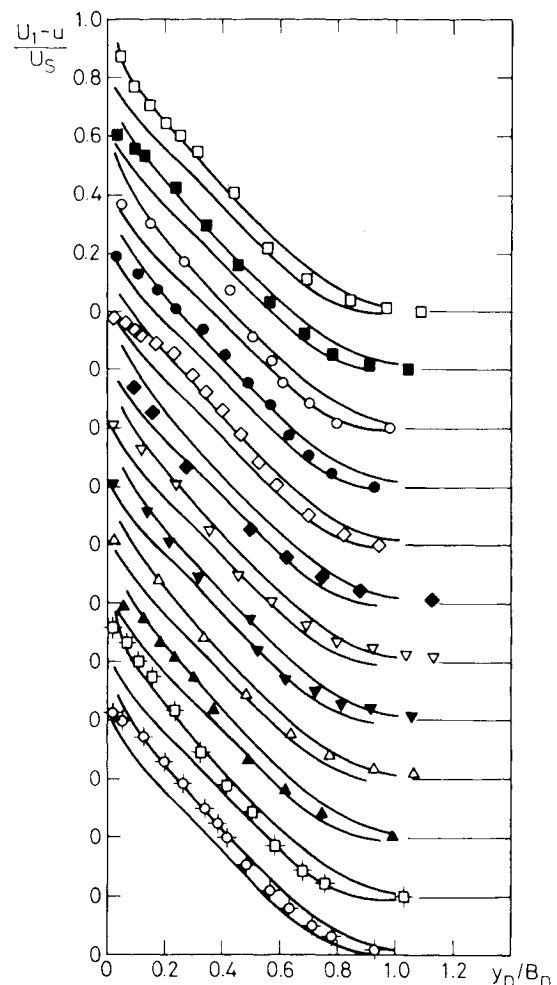


Fig. 8 Mean velocity similarity in two detached layers (data of Delery<sup>22</sup>): — scatter limits for attached data. Flow b:  $\square$  profile 41,  $\blacksquare$  profile 52,  $\circ$  profile 64,  $\bullet$  profile 86,  $\diamond$  profile 109. Flow c:  $\blacklozenge$  profile 35,  $\nabla$  profile 44,  $\triangledown$  profile 54,  $\Delta$  profile 73,  $\blacktriangle$  profile 92,  $\oplus$  profile 111,  $\odot$  profile 131.

because it is valid for detaching, detached, and reattaching layers, as well as for attached layers in medium-to-strong adverse pressure gradient; complementary because near detachment, where the region of validity of the logarithmic law of the wall contracts, the defect law description correspondingly extends down toward the wall. Together, the two laws can provide similarity descriptions that cover the entire positive mean velocity flowfield of a separating layer.

This constitutes a useful advance but to develop a model for separating flowfields a similarity relation for the backflow is required. A similarity description for the reversed flowfield is, however, problematical. Simpson et al.<sup>2,19</sup> and Thompson and Whitelaw<sup>11</sup> have tried a variety of length and velocity scales in an attempt to obtain a collapse of existing backflow data. Wall scales ( $\nu/u_\tau, u_\tau$ ) and outer flow scales ( $U_1, \delta$ ) have been tried by themselves, in combination schemes as well as in zonal descriptions. None satisfactorily correlate the whole backflow profile. Because Simpson et al.<sup>3</sup> found the outer large-scale eddies played a major role in producing the backflow, the author tried to use the outer flow variables  $U_s$  and  $B$  as similarity parameters, but

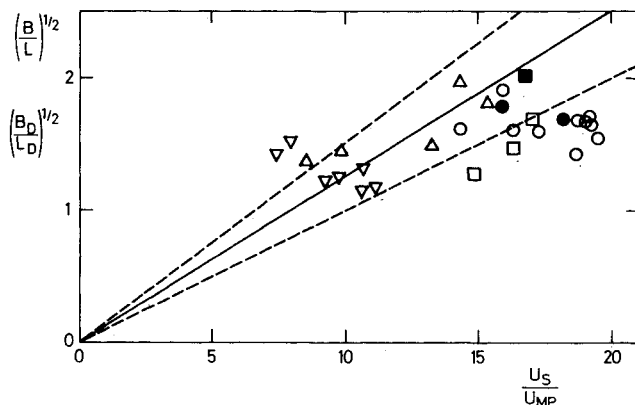


Fig. 9 Integral similarity parameters for detaching layers: solid symbols distinguish attached profiles immediately preceding the detachment layers; solid symbols distinguish attached profiles immediately preceding the detachment zone;  $\square$ ,  $\blacksquare$  Simpson et al.,<sup>1</sup>  $\circ$ ,  $\bullet$  Simpson et al.,<sup>2</sup>  $\triangle$  flow b, Delery<sup>22</sup>  $\nabla$  flow c, Delery<sup>22</sup>; — Eq. (3); --- scatter limits for attached data.

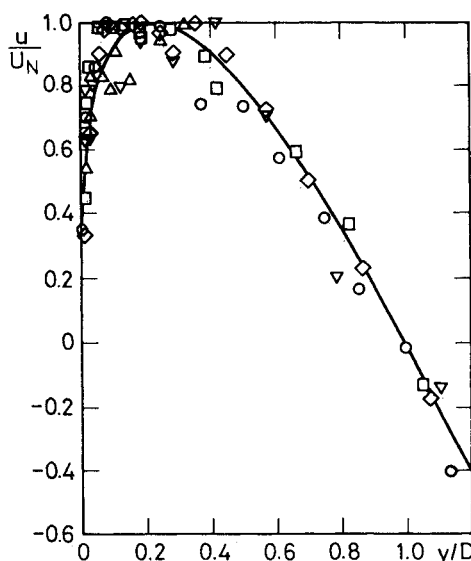


Fig. 10 Mean backflow similarity for data measured with laser doppler anemometry. Data of Simpson et al.<sup>2</sup>:  $\triangle$   $x=139$ ,  $\diamond$   $x=1.44$ ,  $\square$   $x=1.56$ ,  $\circ$   $x=172$ ; data of Simpson et al.<sup>1</sup>  $\nabla$   $x=165$ , — faired in line only.

without success. The reason for the failure of all these schemes probably lies in the fact that the available mean reversed flow data are, in fact, transition data containing significant, but different, components of forward flow. In Simpson's data, the intermittency  $\gamma_p$  near the wall decreases from 0.5 to 0.05 (for flow well downstream of detachment). In addition,  $\gamma_p$  varies across any backflow profile because  $\gamma_p$  is, by definition, 0.5 on the  $u=0$  streamline, but is much lower (0.35–0.05) near the maximum reverse velocity.<sup>2</sup> Thus, what we are trying to analyze are a series of mean velocity profiles containing very different proportions of forward flow, which must scale differently from reversed flow. Such data make the discovery of an underlying similarity for "pure" reversed flow very difficult indeed. It is important to realize, however, that pure backflow ( $\gamma_p=0$  for all  $y$ ) cannot exist because  $\gamma_p$  must by definition be 0.5 on the dividing streamline, which implies a nonzero value for  $\gamma_p$  to some depth below the dividing streamline.

However, the mean reversed profiles do display some similarity using local flow variables. Simpson et al.<sup>2</sup> have demonstrated a fair overall correlation using as scales the maximum reversed velocity  $U_N$  and its distance from the wall. However, as the mean reversed velocity profiles are rather flat near their maxima, Simpson's length scale is difficult to determine accurately, particularly as the backflow measurements tend to have large scatter. The correlations can be improved slightly by using the local backflow thickness  $D$  for the length scale, as this length can be determined quite accurately. This is illustrated in Fig. 10 for data obtained with laser Doppler anemometry. The line faired through these results also correlates quite well with data obtained using intrusive instrumentation (pitot tubes, hot wires), but, as expected, there is considerably more scatter for the intrusive instrumentation data.<sup>24</sup>

### Separation Criteria

In the search for a reliable method of predicting boundary-layer separation, it has been postulated by several authors that turbulent layers detach with a universal mean velocity profile shape.<sup>25–27</sup> Not surprisingly, the proposed "universal" profiles vary in definition. However, as the experimental data of separation have historically been quite poor, it has always been difficult to critically evaluate the different proposals. Part of the problem has been that experimentalists used a variety of definitions of separation and a range of methods of differing sensitivities to detect it. Probably the best empirical study is by Kline et al.,<sup>10</sup> who carefully selected the best data produced over the last 40 years and inferred from it the points at which flows were "intermittently" and "fully" separated. These separation points were related to an empirical profile description that led to separation criteria. The final correlation showed a reasonable level of scatter.

Other separation criteria have been proposed that are not empirically based. These are derived by considering a similarity description that applies to attached flow and taking it to the mathematical limit of separation. Coles<sup>7</sup> observed that, in layers approaching "separation," the vertical extent of the logarithmic region decreased and thus proposed as a separation condition (and as a reattachment condition as well) that the logarithmic region disappeared completely at  $c_f^*=0$ , leaving a pure (Coles') wake profile. This criterion implies a detachment form factor  $H$  of 4.2. A review of data now available shows this value to be too high.

In a similar manner, Simpson et al.<sup>1</sup> and Schofield<sup>17</sup> considered the Schofield-Perry description of the mean flow profile and showed mathematically that at detachment, where  $c_f^*=0$ , the logarithmic region disappears, the half-power region extends to the wall, and  $U_s/U_1=1.0$ . This implies a form factor for the layer at detachment of 2.4. Experimental data show this to be too low. The problem with this approach is in the assumption that the outer similarity

extends right down to the wall at detachment. In fact, a very thin wall layer still exists at detachment (see Figs. 1 and 2), which contains mean velocities as high as  $U_1/20$ . Due to high levels of intermittent flow reversal, the mean similarity of this thin layer is uncertain and has not yet been determined. However, its existence means that the outer similarity region always stops a little above the wall and thus the wall condition ( $u=0$ ,  $y=0$ ) cannot be used to mathematically determine the mean profile shape at detachment. These two examples illustrate the problems of using the mean flow immediately adjacent to the wall to derive a separation criterion. As the results of Simpson et al.<sup>3</sup> show that the outer large-eddy structure is the dominant feature of detaching layers and as this outer flow is described by the Schofield-Perry similarity, it is proposed to use this to develop a new detachment criterion.

Before we do this, it is worth considering the arguments as to why there should exist a universal mean profile shape at detachment. The detachment point of most practical interest is "transitory detachment," where the wall flow spends equal time moving up- and downstream. This point will coincide with the detachment point ( $\tau_w=0$ ) unless the wall shear is of different magnitudes during the up- and downstream phase.<sup>†</sup> At transitory detachment, Simpson's results show  $\gamma_p$  is near 0.5 for a significant distance out from the wall ( $\delta/12$ ), which implies that on a mean basis the fluid in this region is nearly stagnant. Thus, the wall will not play its usual important role in mean flow development, as the usual wall turbulence cannot be generated if the mean velocity near the wall is zero. Another way of saying this is that, at transitory detachment, the upstream wall layer has contracted down to a negligible thickness and the mean reversed flow has not, as yet, developed any depth. With this reduction in wall layer thicknesses, the outer flow turbulence extends its influence closer to the wall than at any other point in the layer. This is illustrated in Fig. 2 where correlation with outer mean flow similarity extends down to within  $0.01\delta$  from the wall at the last attached profile ( $x=134.5$ ). (Corresponding values of  $y/\delta^*$  for subsequent profiles are higher, although they appear with lower  $y_D/D$  values in Fig. 2.) Now if the Schofield-Perry similarity extends down very close to the wall and the mean velocity near the wall is set by the turbulence condition  $\gamma_p=0.5$ , then, because the Schofield-Perry similarity profiles are a one-parameter family, there will be a unique detachment profile with a single value of velocity ratio to specify it. The recent data of Simpson, shown in Fig. 11, suggest the value of  $U_s/U_1$  corresponding with  $\gamma_p=0.5$  near the wall is  $1.2 \pm 0.05$ . Earlier work of Simpson et al.<sup>1</sup> is less helpful, as there are only four data points in the detachment zone, but they do suggest a slightly lower value. As Simpson's detailed laser anemometer measurements for  $\gamma_p$  near separation are unique, supporting evidence for a universal separation profile must rest on mean flow measurement. A selection of data from well-documented nominally two-dimensional separating layers is shown in Figs. 12 and 13. A different (mean flow) criterion of detachment is used in the two figures, but in each case a plausible argument can be advanced to relate each of them to the  $\gamma_p=0.5$  condition. Figure 12 shows wall shear stress determined from the mean flow<sup>‡</sup> plotted vs the similarity velocity ratio  $U_s/U_1$ . As noted previously, the position of  $\tau_w=0$ , the detachment point, will be coincident with the position at which  $\gamma_p=0.5$  or very close to it. The error in assuming the coincidence of these points will be

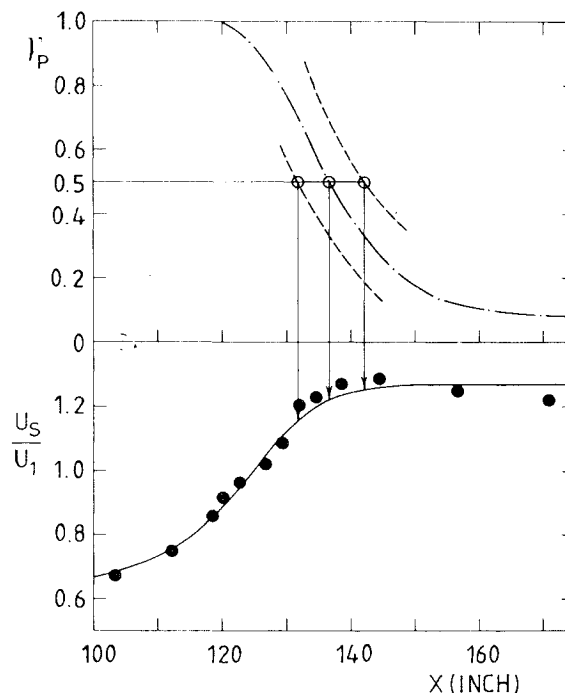


Fig. 11 Wall flow intermittency vs mean profile similarity vector ratio (data of Simpson et al.<sup>2</sup>) • mean flow velocity ratio, — faired in curve through mean flow points — — faired in curve for the intermittency distribution given in Simpson et al.,<sup>2</sup> — — scatter limits for the intermittency data.

smaller than the error involved in extrapolating the wall shearing stress to zero. Figure 12 presents data from a range of layers: separating, near separating, and reattaching, on rough as well as smooth walls. The distributions extrapolate to zero within the range  $1.15 < U_s/U_1 < 1.25$  except perhaps for the reattaching layers after a separation bubble, where the value appears slightly higher, perhaps  $1.2 < U_s/U_1 < 1.3$ . The higher value of  $U_s/U_1$  for a reattaching profile is consistent with the profile development to be expected after detachment. The removal of the wall restraint after detachment allows the mean strain near the profile origin ( $y_D=0$ ) to relax to a lower value that matches the reversed flow mean strain. This implies that the velocity scale  $U_s$ , which is in fact a "wall slip" velocity (see Ref. 14) increases. An additional effect of detachment is that the turbulence structure develops some little way toward the (higher) levels appropriate for a two-dimensional free shear layer. Troutt et al.<sup>28</sup> found that turbulence in a separated layer develops rapidly toward that of a free shear layer which has a higher turbulence. The development in the turbulence structure reported by Simpson et al.<sup>2</sup> for a separating layer was also in the same direction.

The evidence of Fig. 12 is supportive but, of course, weakened by the fact that the extrapolations rely heavily on the points nearest detachment where the standard law of the wall, used to determine the wall shear, is most in doubt due to the reversed flow contamination of the mean velocity measurements. An alternative approach that considers the behavior of the whole layer rather than just the wall region would, therefore, seem attractive. Such an approach has been attempted previously (e.g., Kline et al.<sup>10</sup>), but these analyses have encountered difficulties in handling the reversed flow component of the profiles, which can vary in depth from flow to flow. Such problems can be circumvented by using the extended validity of Schofield-Perry similarity with its origin on the dividing streamlines for flow downstream of detachment. Equation (1) can be integrated

<sup>†</sup>In attached layers, Sandborn's<sup>25</sup> results suggest that  $\tau_w$  does have different magnitudes during up- and downstream phases. However, the difference appears to decrease as separation is approached. Furthermore, Simpson et al.<sup>23</sup> have shown that near the wall the distribution of velocity fluctuations is Gaussian.

<sup>‡</sup>From different methods, but all assuming the validity of the law of the wall.

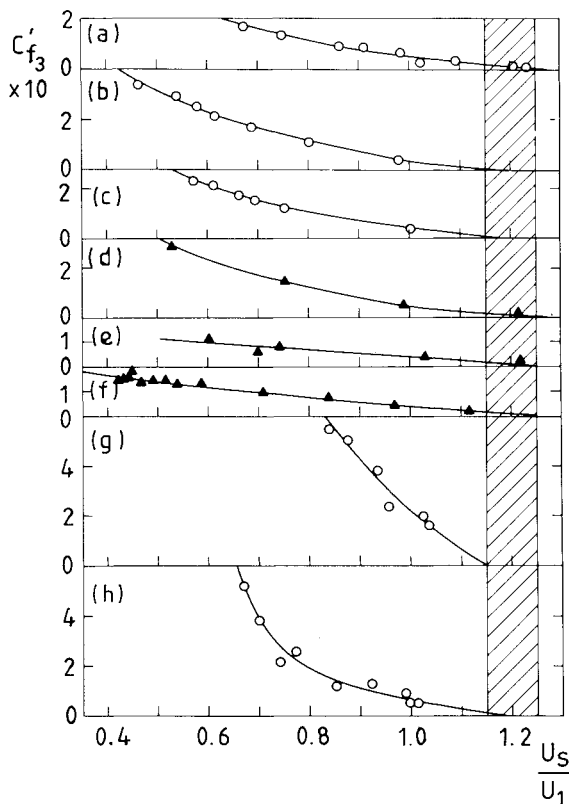


Fig. 12 Wall shear stress distributions of detaching and reattaching layers:  $\circ$  profiles approaching detachment,  $\blacktriangle$  profiles after reattachment, shaded area is the detachment condition  $1.15 < U_s/U_1 < 1.25$ . Data of: a) Simpson et al.<sup>2</sup>; b) Fairlie,<sup>31</sup> flow I; c) Fairlie,<sup>31</sup> flow II; d) Seddon,<sup>20</sup> basic interaction; e) Seddon,<sup>20</sup> modified interaction; f) Schofield<sup>21</sup>; g) Perry et al.,<sup>32</sup> KI-I series; h) Perry et al.,<sup>32</sup> DII-II series.

across the layer to yield

$$H, H_D = (1 - 0.58 U_s/U_1)^{-1} \quad (4)$$

where the notation  $H, H_D$  implies that integration commences on the dividing streamline for profiles after detachment. This equation implies that there is a single  $H, H_D$  vs  $U_s/U_1$  locus for all attached and separated layers. With the assumption of a universal detachment profile, it is further implied that detachment always occurs at the same point on this locus. Equation (4) is somewhat inaccurate because its derivation assumes Eq. (1) is valid all the way down to the wall, whereas Eq. (1) can be quite inaccurate at small  $y/B$  if there is a thick logarithmic region. For this reason Eq. (4) is shown in Fig. 12 as a pair of parallel lines (corresponding to a variation of 10%) against which the loci of a range of separating and reattaching layers are compared. The loci of separating layers all move uniformly up the curve. Reattaching layers move uniformly down the curve. The postulate being advanced here is that detachment in a separating layer occurs when  $U_s/U_1$  has a value of 1.20, which corresponds to the layer developing a profile with a form factor of approximately 3.3. The detachment criterion used for the data of Fig. 13 is simply the first appearance of mean reversed flow near the wall, which indicates that  $\gamma_p$  has dropped below 0.5. The methods used to determine this first appearance of mean reversed flow are questionable in all of the data except Simpson's. Physically intrusive techniques to measure the mean flow near the wall (pitot tubes, hot wires) are subject to large uncertainties when used in flow regions with substantial periods of reversed flow. Similarly, the alternative technique of surface flow visualization is not accurate because the variants (tufts, oil films, fine dust) are

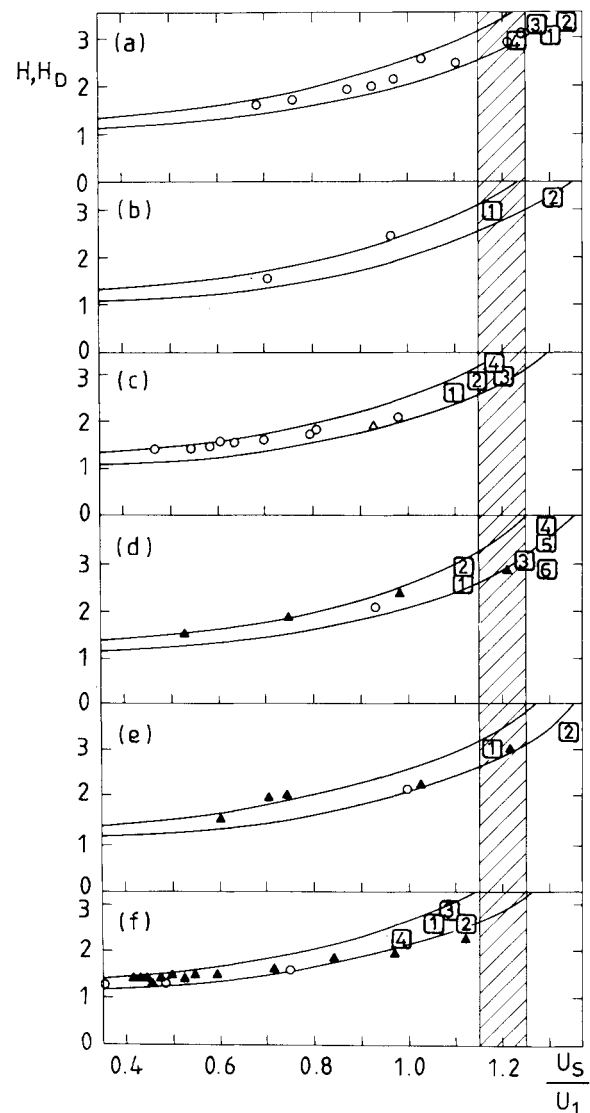


Fig. 13 Form factor vs velocity ratio for detaching and reattaching layers:  $\circ$  profiles approaching detachment,  $\blacktriangle$  profiles after reattachment, numbers in squares denote detached profiles with the number giving the downstream order of the profiles, shaded area is the detachment condition  $1.15 < U_s/U_1 < 1.25$ , —, Eq. 4  $\pm 10\%$ . Data of a) Simpson et al.,<sup>2</sup> b) Simpson et al.,<sup>1</sup> c) Fairlie,<sup>31</sup> flow I, d) Seddon,<sup>19</sup> basic interaction, e) Seddon,<sup>19</sup> modified interaction, f) Schofield.<sup>21</sup>

likely to respond to different levels of intermittent backflow and these levels are unknown. The lighter indicators (e.g., dust) respond to the more probable wall velocity and are thus in error if the results differ from the mean wall velocity (see Ref. 29). For the data of Fig. 13, the first detached profile in a separating layer (and the last detached profile in a reattaching layer) was determined from the mean flow profile, the wall flow visualization pattern, or a combination of the two. The resulting evidence, although scattered, supports the proposition that detachment occurs when the layer has developed a profile shape defined by a similarity velocity ratio near 1.2. The data for reattaching flows suggest that reattachment occurs at a slightly higher velocity ratio than detachment, which is consistent with the skin-friction data of Fig. 12.

Separating boundary layers are difficult flows in which to obtain reliable or consistent measurements. In view of this, the data presented here conform with the proposed detachment criterion with an encouraging consistency. A more accurate value of the detachment velocity ratio is unlikely to be much different from 1.2.



### Conclusions

Schofield<sup>17</sup> saw as an advantage of the Schofield-Perry similarity scales that they did not need modification to describe "near-separating layers in very severe pressure gradients." This advantage has now been extended to include separating, separated, and reattaching layers at the cost of relocating the profile origin for similarity onto the dividing streamline in the case of fully detached layers. It should be noted, however, that the position of the zero-velocity streamline cannot be predicted and that, as the similarity relations were formulated with two-dimensional data, they are somewhat inaccurate for detached flows having significant three-dimensional flow components. Nevertheless, for the rather different layers analyzed in this paper, the Schofield-Perry defect law provides a description of the outer 90–95% of all profiles with an acceptable accuracy. The similarity does not extend very close to the wall in an attached layer nor right down to the dividing streamline in a detached layer. The inner flows that match the outer large-scale flow to the zero mean velocity "wall" condition show a smooth transition in shape from fully attached to fully detached. However, the smooth transition is more apparent than real, because forward and reversed flows near the wall have quite different velocity distributions and the apparently smooth transition is a result of mean measurements containing gradually increasing proportions of backflow as detachment develops with the distance downstream. Near the wall, the proportion of time spent in forward flow ( $\gamma_p$ ) continues to decrease for a considerable distance downstream of the detachment point and, in addition, there must be a large vertical variation in  $\gamma_p$ . Thus, a fundamental problem in finding a universal similarity description for the backflow exists in that the mean backflow profiles contain varying proportions of forward flow in their measurements. This is probably the reason why none of the plausible similarity schemes proposed by various authors have been successful. A small improvement in the Simpson et al. local similarity scheme<sup>2</sup> is possible, if we use the total backflow thickness as the length scale in conjunction with the maximum reverse velocity to collapse the data.

The extended range of validity of Schofield-Perry similarity implies a unique progression of two-dimensional mean velocity profile shapes up to and through separation. The proposal that two-dimensional detachment occurs at the same point in this progression has good experimental support. The universal mean velocity profile at detachment has a similarity velocity ratio near 1.20, which implies a form factor of 3.3. It is interesting to note that a practical rule of thumb used for some time by designers of aerodynamic devices is that "turbulent boundary layers separate when the form factor exceeds three." After detachment, the forward flowing outer portion of the layer shows some additional development up to a velocity ratio near 1.25. Although there is less evidence to support this conclusion, it appears that layers reattach with a universal mean velocity profile. The velocity ratio at reattachment is probably that for fully detached layers and is therefore slightly larger than the value for a transitory detachment profile. The existence of universal profiles at detachment and reattachment may be attributed to very thin wall layers in these regions. Here, the outer large-scale turbulence structure that determines the outer mean profile also largely determines the flow near the wall.

### References

- <sup>1</sup>Simpson, R. L., Strickland, J. H., and Barr, P. W., "Features of a Separating Turbulent Boundary Layer in the Vicinity of Separation," *Journal of Fluid Mechanics*, Vol. 79, March 1977, pp. 553-594.
- <sup>2</sup>Simpson, R. L., Chew, Y. T., and Shivaprasad, B. G., "The Structure of a Separating Turbulent Boundary Layer, Part 1: Mean Flow and Reynolds Stresses," *Journal of Fluid Mechanics*, Vol. 113, Dec. 1981, pp. 23-51.
- <sup>3</sup>Simpson, R. L., Chew, Y. T., and Shivaprasad, B. G., "The Structure of a Separating Turbulent Boundary Layer, Part 2: Higher-Order Turbulence Results," *Journal of Fluid Mechanics*, Vol. 113, Dec. 1981, pp. 53-73.
- <sup>4</sup>Simpson, R. L., Shivaprasad, B. G., and Chew, Y. T., "The Structure of a Separating Turbulent Boundary Layer, Part 4: Effects of Periodic Free-Stream Unsteadiness," *Journal of Fluid Mechanics*, Vol. 127, Feb. 1983, pp. 219-261.
- <sup>5</sup>Simpson, R. L., "A Review of Some Phenomena in Turbulent Flow Separation," *Journal of Fluids Engineering*, Vol. 103, Dec. 1981, pp. 520-533.
- <sup>6</sup>Kline, S. J., Ferziger, J. H., and Johnston, J. P., "Calculation of Turbulent Shear Flows: Status and Ten-Year Outlook," *Journal of Fluids Engineering*, Vol. 100, March 1978, pp. 3-5.
- <sup>7</sup>Coles, D., "The Law of the Wake in a Turbulent Boundary Layer," *Journal of Fluid Mechanics*, Vol. 1, July 1956, pp. 191-226.
- <sup>8</sup>Coles, D., "The Young Person's Guide to the Data," *Proceedings of AFSOR-IFP-Stanford Conference on Computation of Turbulent Boundary Layers*, Vol. II, Stanford University, CA, 1968, pp. 1-54.
- <sup>9</sup>Coles, D. E. and Hirst, E. A., "A Computation of Turbulent Boundary Layers," *Proceedings of AFSOR IFP-Stanford Conference on Computation of Turbulent Boundary Layers*, Vol. II, Compiled data, Stanford University, CA, 1968.
- <sup>10</sup>Kline, S. J., Bardina, J. G., and Strawn, R. C., "Correlation of the Detachment of Two-Dimensional Turbulent Boundary Layers," *AIAA Journal*, Vol. 21, Jan. 1983, pp. 68-73.
- <sup>11</sup>Thompson, B. E. and Whitelaw, J. H., "Characteristics of a Trailing-Edge Flow with Turbulent Boundary-Layer Separation," *Journal of Fluid Mechanics*, Vol. 157, Aug. 1985, pp. 305-326.
- <sup>12</sup>Wei, Q.-D. and Sato, H., "An Experimental Study of the Mechanism of Intermittent Separation of a Turbulent Boundary Layer," *Journal of Fluid Mechanics*, Vol. 143, June 1984, pp. 153-172.
- <sup>13</sup>Bradshaw, P., "The Turbulence Structure of Equilibrium Boundary Layers," *Journal of Fluid Mechanics*, Vol. 29, Sept. 1967, pp. 625-645.
- <sup>14</sup>Schofield, W. H. and Perry, A. E., "The Turbulent Boundary Layer as a Wall Confined Wake," Aeronautical Research Labs, Australian Dept. of Defence, Mech. Eng. Rept. 134, Feb. 1972.
- <sup>15</sup>Perry, A. E. and Schofield, W. H., "Mean Velocity and Shear Stress Distributions in Turbulent Boundary Layers," *The Physics of Fluids*, Vol. 16, No. 12, Dec. 1973, pp. 2068-2074.
- <sup>16</sup>Samuel, A. E. and Joubert, P. N., "A Boundary Layer Developing in an Increasingly Adverse Pressure Gradient," *Journal of Fluid Mechanics*, Vol. 66, Nov. 1974, pp. 481-505.
- <sup>17</sup>Schofield, W. H., "Equilibrium Boundary Layers in Moderate to Strong Adverse Pressure Gradients," *Journal of Fluid Mechanics*, Vol. 113, Dec. 1981, pp. 91-122.
- <sup>18</sup>Clauser, F. H., "Turbulent Boundary Layers in Adverse Pressure Gradients," *Journal of the Aeronautical Sciences*, Vol. 21, Feb. 1954, pp. 91-109.
- <sup>19</sup>Simpson, R. L., "A Model for the Backflow Mean Velocity Profile," *AIAA Journal*, Vol. 21, Jan. 1983, pp. 142-143.
- <sup>20</sup>Seddon, J., "The Flow Produced by Interaction of a Turbulent Boundary Layer with a Normal Shock Wave of Strength Sufficient to Cause Separation," Ministry of Technology, Aeronautical Research Council, Reports and Memoranda, R&M 3502, 1967.
- <sup>21</sup>Schofield, W. H., "Turbulent Boundary Layer Development in an Adverse Pressure Gradient After an Interaction with a Normal Shock Wave," *Journal of Fluid Mechanics*, Vol. 154, May 1985, pp. 43-62.
- <sup>22</sup>Delery, J. M., "Investigation of Strong Shock Turbulent Boundary Layer Interaction in 2-D Transonic Flows with Emphasis on Turbulence Phenomena," AIAA Paper 81-1245, June 1981.
- <sup>23</sup>Simpson, R. L., Strickland, J. H., and Barr, P. W., "Laser and Hot Film Anemometer Measurements in a Separating Turbulent Boundary Layer," Southern Methodist University, Dallas, TX, Tech. Rept. WT-3, Sept. 1974.
- <sup>24</sup>Schofield, W. H., "On Separating Turbulent Boundary Layers," Aeronautical Research Labs, Australian Dept. of Defence, Mech. Eng. Rept. 162, Sept. 1983.
- <sup>25</sup>Sandborn, V. A., "Boundary Layer Separation and Reattachment," *Fluid Mechanics, Acoustics and Design of Turbomachinery*, NASA SP-304, Pt. 1, 1970, p. 279.
- <sup>26</sup>Sandborn, V. A. and Kline, S. J., "Flow Models in Boundary-Layer Stall Inception," *Journal of Basic Engineering*, Vol. 83, Sept. 1961, pp. 317-327.

<sup>27</sup>Jacob, K., "Computation of Separated Incompressible Flow Around Airfoil and Determination of Maximum Lift," *Zeitschrift für Flugwissenschaften*, Vol. 17, July 1969, pp. 221-230.

<sup>28</sup>Troutt, T. R., Scheelke, B., and Norman, T. R., "Organized Structures in a Reattaching Separated Flow Field," *Journal of Fluid Mechanics*, Vol. 143, June 1984, pp. 413-427.

<sup>29</sup>Sandborn, V. A., "Surface Shear Stress Fluctuations in Turbulent Boundary Layers," *Proceedings of Second Symposium on Turbulent Shear Flows*, Bradbury, London, 1979, p. 4.1.

<sup>30</sup>Perry, A. E. and Fairlie, B. D., "A Study of Turbulent Boundary-Layer Separation and Reattachment," *Journal of Fluid Mechanics*, Vol. 69, June 1975, pp. 657-672.

<sup>31</sup>Fairlie, B. D., Ph.D. Thesis, University of Melbourne, Australia, 1973.

<sup>32</sup>Perry, A. E., Schofield, W. H., and Joubert, P. N., "Rough Wall Turbulent Boundary Layers," *Journal of Fluid Mechanics*, Vol. 37, June 1969, pp. 383-413.

## *From the AIAA Progress in Astronautics and Aeronautics Series . . .*

### **AERO-OPTICAL PHENOMENA—V. 80**

*Edited by Keith G. Gilbert and Leonard J. Otten, Air Force Weapons Laboratory*

This volume is devoted to a systematic examination of the scientific and practical problems that can arise in adapting the new technology of laser beam transmission within the atmosphere to such uses as laser radar, laser beam communications, laser weaponry, and the developing fields of meteorological probing and laser energy transmission, among others. The articles in this book were prepared by specialists in universities, industry, and government laboratories, both military and civilian, and represent an up-to-date survey of the field.

The physical problems encountered in such seemingly straightforward applications of laser beam transmission have turned out to be unusually complex. A high intensity radiation beam traversing the atmosphere causes heat-up and breakdown of the air, changing its optical properties along the path, so that the process becomes a nonsteady interactive one. Should the path of the beam include atmospheric turbulence, the resulting nonsteady degradation obviously would affect its reception adversely. An airborne laser system unavoidably requires the beam to traverse a boundary layer or a wake, with complex consequences. These and other effects are examined theoretically and experimentally in this volume.

In each case, whereas the phenomenon of beam degradation constitutes a difficulty for the engineer, it presents the scientist with a novel experimental opportunity for meteorological or physical research and thus becomes a fruitful nuisance!

*Published in 1982, 412 pp., 6×9, illus., \$29.50 Mem., \$59.50 List*

TO ORDER WRITE: Publications Dept., AIAA, 1633 Broadway, New York, N.Y. 10019